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A new model of thermal conductivity for liquids

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Abstract

Using our previous estimation equation of the heat of vaporization for liquids with the residual function method, and with the liquid free volume model as well as the theory of molecular dynamics, a new two-parameter model of thermal conductivity for liquids has been derived. With this new model, the thermal conductivity data for 68 kinds of liquids at 1217 temperature sets were tested. The results are compared with the recent models proposed by Lei et al. [Q. Lie, Y.C. Hou, R. Lin, Chem. Eng. Sci. 52 (1997) 1234; Q. Lie, R. Lin, D. Ni, J. Chem. Eng. Data 42 (1997) 971], Klass et al. [D.M. Klaas, D.S. Viswanath, Ind. Eng. Chem. Res. 37 (1998) 2064], and with the Jamieson equation and the modified Riedel equation. The comparison shows that this model is applicable to many kinds of liquids for a relatively wide range of temperature up to the critical point. © 2000 Elsevier Science S.A. All rights reserved.

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1. Introduction

Thermal conductivity data of liquids are important in the design of chemical engineering, but it is difficult to accurately measure them. Even for the same liquid at the same temperature, the experimental thermal conductivity measured by different investigators shows considerable disagreement. Since theoretical methods have not yielded sufficiently accurate and simple expressions for calculating the thermal conductivity of liquids, correlation based on empirical or semi-theoretical method is widely employed over limited ranges of temperature [1–4]. Recent correlations are due to Jamieson [5,6], Teja and co-workers [7–9], Klaas and Viswanath [10], and Lei and co-workers [11,12]. However, many of these methods have some limitations, suggesting that a new model of thermal conductivity for liquids is wanted, which has some theoretical basis and is applicable to more kinds of liquids for a wider range of temperature.

In this paper, a new model of thermal conductivity for liquids, which may be used over wide ranges of temperature, even up to the critical temperature, is proposed on the basis of the authors' estimation equation of the heat of vaporization for liquids with the residual function method, and with the liquid-free volume model as well as the theory of molecular dynamics. The results are compared with the recent methods proposed by Lei et al. and Klaas et al., and with the Jamieson equation and the modified Riedel equation.

2. Derivation of the new model

Based on the absolute reaction rate theory of Eyring [13] and the free volume theory [14], Zhang and Liu [15] showed that the liquid viscosity is inversely proportional to the product of the probability P_E of containing activation energy E_a for a molecule and the probability P_V of possessing proper free volume around the molecule. On the basis of the ideas of Zhang et al. and the fact that the influences of temperature on thermal conductivity and viscosity have the same trend [16,17], referring to the thought of Teja and co-workers [9,18–20] that thermal conductivity and viscosity have analogous expressions, we conclude that thermal conductivity is inversely proportional to the product of P_E and P_V . Thus,

$$
\lambda^{-1} = a P_E P_V \tag{1}
$$

where P_E and P_V are expressed as Eyring's [13,21] and Ertl's [22] relations, respectively:

$$
P_E = b \exp\left(-\frac{E_a}{RT}\right) \tag{2}
$$

$$
P_V = eV_I^n \tag{3}
$$

According to [13,15], the molar activation energy *E*a, and the liquid lattice energy E_c are in proportion as follows:

$$
E_{\rm a} = dE_{\rm c} \tag{4}
$$

By introducing Eqs. (2)–(4) into Eq. (1) and taking natural logarithm in Eq. (1), the following is obtained:

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$$
\ln \lambda = -\ln(abe) - n \ln V_f + \frac{dE_c}{RT}
$$
 (5)

Assuming that Eq. (5) is applicable to the vapor phase of a vapor–liquid equilibrium, and the vapor phase is dilute gases, then $E_c^V \to 0$, $V_f^V \to V$ [23]. Thus,

$$
\ln \lambda^V = -\ln(abe) - n \ln V \tag{6}
$$

By subtracting Eq. (6) from Eq. (5), the following is obtained:

$$
\ln \frac{\lambda}{\lambda V} = -n \ln \frac{V_f}{V} + \frac{dE_c}{RT}
$$
 (7)

According to the theory of chemical engineering thermodynamics, chemical potentials are equal at vapor–liquid equilibrium and can be expressed by configuring partition function *Q* as follows [23]:

$$
\left(\frac{\partial \ln Q^{\mathcal{L}}}{\partial N}\right)_{T,V} = \left(\frac{\partial \ln Q^{\mathcal{V}}}{\partial N}\right)_{T,V}
$$
(8)

and

$$
Q^{\mathcal{L}} = V_{\mathbf{f}}^{N} \exp\left(-\frac{E_{\mathbf{c}}}{RT}\right) \tag{9}
$$

$$
Q^{\mathcal{V}} = \frac{V^N}{N!} \tag{10}
$$

Eqs. (9) and (10) are substituted into Eq. (8) and the Stirling approximation is adopted, with $V_f = Nv_f$ and $E_c = N^2 f(V,$ *T*) [24]; the following relation is obtained:

$$
\ln\frac{V_{\rm f}}{V} = \frac{2E_{\rm c}}{nkT} \tag{11}
$$

Substituting Eq. (11) into Eq. (7), we get

$$
\ln \frac{\lambda}{\lambda^{\rm V}} = \frac{BE_{\rm c}}{RT} \tag{12}
$$

The liquid lattice energy E_c is approximately equal to the vaporization energy U_V [24] and the latter is related to the vaporization heat L_V as follows [1]:

$$
E_{\rm c} \approx U_{\rm V} = L_{\rm V} - RT \tag{13}
$$

According to the theory of molecular dynamics [1,2,4], the thermal conductivity of dilute gases is [11]

$$
\lambda^{\rm V} = \frac{C(T/M)^{1/2}}{V_{\rm c}^{2/3} \Omega_{\rm V}}\tag{14}
$$

where the collision integration Ω_V is expressed by the Neufeld–Janzen–Aziz equation [25] as follows:

$$
\Omega_{\rm V} = 0.52487 \exp(-0.97369 T_{\rm r}) + 2.16178
$$

× exp(-3.07001 T_{\rm r}) + 1.1223 T_{\rm r}^{-0.14874} (15)

By taking the natural logarithm in Eq. (14) and defining

$$
q = \Omega_V V_c^{2/3} \left(\frac{T}{M}\right)^{-1/2} \tag{16}
$$

the following is obtained:

$$
\ln \lambda^V = \ln C - \ln q \tag{17}
$$

Introducing Eqs. (13) and (17) into Eq. (12) and arranging it, we get

$$
\ln(\lambda q) = A + \frac{BL_V}{RT}
$$
 (18)

We [26,27] have proposed the new equation to estimate the heat of vaporization for pure liquids based on the statistical thermodynamics theory as follows:

$$
L_V = \left(\frac{RT_c SW}{P_c}\right) \left(\frac{P_c \ln P_c}{\theta - 1} - C^0 - \omega C'\right) \tag{19}
$$

where

$$
S = (10 + 3T_{r} - 2T_{r}^{2}) \left(\frac{\theta^{2}}{10\theta^{2} + 3\theta - 2}\right)
$$

\n
$$
W = \left(\frac{\theta - (T/T_{b})}{\theta - 1}\right)^{0.38}
$$

\n
$$
C^{0} = 1.097\theta^{1.6} - 0.083
$$

\n
$$
C' = 0.8944\theta^{4.2} - 0.139
$$

\n
$$
\theta = \frac{T_{c}}{T_{b}}
$$

\n
$$
T_{r} = \frac{T}{T_{c}}
$$

The acentric factor in Eq. (19) is calculated by the Lee–Kesler equation [28].

Combining Eqs. (18) and (19), the new equation to calculate the thermal conductivity of pure liquids is obtained as follows:

$$
\ln(\lambda q) = A + B \frac{SWT_r^{-0.5}}{P_c} \left(\frac{P_c \ln P_c}{\theta - 1} - C^0 - \omega C' \right) \tag{20}
$$

3. Results and discussion

By use of the least squares method, the two characteristic parameters *A* and *B* of a liquid in Eq. (20) can be determined from experimental data over the investigated temperature range. The values of *A* and *B* for some liquids are given in Table 1.

With the parameters *A* and *B*, we can use the most fundamental physical property data comprising T_b, T_c, P_c and *V*^c to calculate the thermal conductivity of some substances at any temperature according to Eq. (20). The thermal conductivity of 68 pure liquids including paraffins, olefins, alkynes, cycloparaffins, aromatics, alcohols, phenol, ethers, aldehydes, ketones, esters, organic acids, organic halides, organic nitrogen compounds and inorganic compounds were calculated from Eq. (20) over wide ranges of temperature,

Liquids	Temperature range, T_r	No. of points	A	$B \times 10$	ARD(%)					Refs.
					New model	Jamieson	Riedel	Klaas et al.	Lei et al.	
Sulfur dioxide	$0.46425 - 0.92851$	15	2.2273	1.8890	0.22	1.85	1.56	3.16	2.82	[16, 17, 36]
Hydrogen fluoride	$0.63470 - 1.0000$	9	1.7282	1.9170	1.23	3.79	1.62	3.62	3.29	$[17]$
Hydrogen chloride	0.53342-0.93962	10	2.001	2.5376	2.30	4.09	4.34	4.19	4.17	$[17]$
Hydrogen sulfide	0.57028-0.91116	9	1.0464	3.3193	2.07	3.94	2.67	5.52	4.92	$[17]$
Ammonia	0.59117-0.96166	38	1.9562	2.2783	1.40	3.86	1.95	3.59	2.69	[16, 17, 34]
Dowtherm J	0.34009-0.94491	9	3.4327	0.8740	1.17	1.54	0.33	0.25	0.55	[17]
Mercury	$0.3574 - 0.59550$	10	1.9630	1.3871	0.53	0.30	0.80	0.43	0.45	$[17]$
Water (0.1 MPa)	0.42198-0.57647	44	3.3653	0.4102	0.68	0.54	0.79	0.66	0.70	[29, 33, 35]
Water (5 MPa)	0.42198-0.57647	11	2.5115	0.4940	0.84	0.67	0.97	0.82	0.76	[33]
Water (20 MPa)	0.42198-0.57647	11	2.5063	0.5147	0.84	0.68	0.99	0.82	0.76	$[33]$
Total 68		1217			1.70	3.43	2.04	5.21	4.09	

Table 1 (*Continued*)

even up to the critical temperature. The results are shown in Table 1. The reference data of thermal conductivity come from Vargaftik [16], Beaton [17], Jamieson [29], Lei et al. [12], Rohsenow [30], Digullio et al. [31] and Cai and others [32–36]. Table 1 also shows the comparison between the new model and other good models, such as the Jamieson equation [4–6], the modified Reidel equation [7] and the recent models proposed by Klaas et al. [10] and Lei et al. [11]. Comparing the calculated data and reference data of 68 pure liquids at 1217 temperature points, the overall average relative deviations of five models are, respectively, 1.70, 3.43, 2.04, 5.21, and 4.09%. The overall average relative deviation is calculated as follows:

$$
ARD = \left(\frac{100}{N_{\rm p}}\right) \sum \left[\frac{|\lambda_{\rm p} - \lambda_{\rm e}|}{\lambda_{\rm e}}\right]
$$
 (21)

Obviously, the new model is the best over the investigated temperature as well as substance range, and the modified Reidel equation takes the second place. The Jamieson equation is applicable to common liquids, but has notable errors when applied to lower paraffin hydrocarbons, some refrigerants, liquefied inert gas, diatomic gas and inorganic gas, especially when temperatures are close to the critical temperature. The model proposed by Klaas et al. can give good results when the temperature range is very limited. However, the calculated error rises sharply when the temperature range is extended, especially up to the critical temperature. Klaas et al., however, pointed out [10] that their model gives good results for the temperature range between the normal melting point and the normal boiling point of a substance. The method of Lei et al. has the same limitation as the model by Klaas et al.

4. Conclusions

Using our previous estimation equation of the heat of vaporization for liquids with the residual function method, and with the liquid free volume model as well as the theory of molecular dynamics, a new two-parameter model of thermal conductivity for liquids has been derived. With this new model, the thermal conductivity data for 68 kinds of liquids at 1217 temperature sets were tested. The results are compared with the recent models proposed by Lei et al., Klaas et al., and with the Jamieson equation and the modified Riedel equation. The comparison shows that the new model, which has a theoretical basis, is applicable to paraffins, olefins, alkynes, cycloparaffins, aromatics, alcohols, phenol, ethers, aldehydes, ketones, esters, organic acids, organic halides, organic nitrogen compounds, refrigerants, liquefied inert gas, inorganic compounds, and so on. The new model can give good results over a wide temperature range, even up to the critical temperature.

5. Nomenclature

Greek symbols

λ thermal conductivity

- Ω_V collision integral
- ω acentric factor

Superscripts

- L saturated liquid phase
- V saturated vapor phase

Subscripts

- b normal boiling point
- c critical point
- e experimental data
- P calculated data
- r reduced property

References

- [1] R.C. Reid, J.M. Prausnitz, B.E. Poling, The Properties of Gases and Liquids, 4th Edition, McGraw-Hill, New York, 1987.
- [2] M. Moetamast, Viscosity and Thermal Conductivity, Masukaza yakata, Tokyo, 1975.
- [3] V.S. Touloukian, P.E. Liley, S.C. Saxena, Thermal Conductivity, Nonmetallic Liquids and Gases, Plenum Press, New York, 1970.
- [4] F.A. Wang, Introduction to Chemical Engineering Data, Chem. Ind. Press, Beijing, 1995.
- [5] D.T. Jamieson, J. Chem. Eng. Data 24 (1979) 244.
- [6] D.T. Jamieson, G. Cartwright, J. Chem. Eng. Data 25 (1980) 199.
- [7] A.S. Teja, P. Rice, Chem. Eng. Sci. 36 (1981) 417.
- [8] A.S. Teja, G. Tardieu, Can. J. Chem. Eng. 66 (1988) 980.
- [9] J.G. Bleazard, A.S. Teja, Ind. Eng. Chem. Res. 35 (1996) 2453.
- [10] D.M. Klaas, D.S. Viswanath, Ind. Eng. Chem. Res. 37 (1998) 2064.
- [11] Q. Lei, Y.C. Hou, R. Lin, Chem. Eng. Sci. 52 (1997) 1243.
- [12] Q. Lei, R. Lin, D. Ni, J. Chem. Eng. Data 42 (1997) 971.
- [13] S. Glasstone, K.J. Laidler, H. Eyring, The Theory of Rate Processes, McGraw-Hill, New York, 1941.
- [14] J.H. Hildebrand, Viscosity and Diffusivity, Wiley, New York, 1977.
- [15] J.H. Zhang, H.Q. Liu, J. Chem. Ind. Eng. (China) 5 (1990) 276.
- [16] N.B. Vargaftik, Tables on the Thermodynamical Properties of Liquids and Gases, Hemisphere, Washington, 1975.
- [17] C.F. Beaton, G.F. Hewitt, Physical Property Data for the Design Engineers, Hemisphere, Washington, 1989.
- [18] T.F. Sun, J. Bleazard, A.S. Teja, J. Phys. Chem. 98 (1994) 1306.
- [19] T.H. Chung, M. Ajlan, L.L. Lee, K.E. Starling, Ind. Eng. Chem. Res. 27 (1988) 671.
- [20] R.P. Chhabra, T. Sridhar, P.H.T. Uhlherr, AIChE J. 26 (1980) 522.
- [21] J.R. Van Wazer, J.W. Lyons, K.Y. Kim, Viscosity and Flow Measurement, Interscience, New York, 1963.
- [22] H. Ertl, F.A.L. Dullien, J. Phys. Chem. 77 (1973) 3007.
- [23] Y. Hu, G.J. Liu, Y.N. Xu, Applied Statistical Thermodynamics, Chem. Ind. Press, Beijing, 1990.
- [24] Y.Q. Tang, Statistical Mechanics and its Applications in Physical Chemistry, Science Press, Beijing, 1979.
- [25] P.D. Neufeld, A.R. Janzen, R.A. Aziz, J. Chem. Phys. 57 (1972) 1100.
- [26] F.A. Wang, W.B. Zhao, C.S. Yang, J. Chem. Ind. Eng. (China) 40 (1989) 489.
- [27] F.A. Wang, Y.L. Jiang, W.C. Wang, D.G. Jiang, Chem. Eng. J. 59 (1995) 101.
- [28] B.I. Lee, M.G. Kes1er, AIChE J. 21 (1975) 510.
- [29] D.T. Jamieson, J.B. Irving, J.S. Tudhope, Liquid Thermal Conductivity, a Data Survey to 1973, Her Majesty's Stationery Office, Edinburgh, 1975.
- [30] W.M. Rohsenow, Handbook of Heat Transfer Fundamentals, 2nd Edition, McGraw-Hill, New York, 1985.
- [31] R.M. Digullio, W.L. McGregor, A.S. Teja, J. Chem. Eng. Data 37 (1992) 242.
- [32] G. Cai, H. Zong, Q. Yu, R. Lin, J. Chem. Eng. Data 38 (1993) 332.
- [33] W.M. Rohsenow, J.P. Hartnett, Handbook of Heat Transfer, McGraw-Hill, New York, 1973.
- [34] M.L. Lu, C. McGreavy, E.K.T. Kam, J. Chem. Eng. Jpn. 30 (1997) 12.
- [35] H. Mensab-Brown, W.A. Wakeham, Int. J. Thermophys. 15 (1994) 647.
- [36] Kagaku Binran, Chem. Soc. Japan, 3rd Edition, Maruzen, Tokyo, 1984, pp. 11–73.